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Fish Harvesting Management Strategies Using Logistic Growth Model

(Strategi Pengurusan Penuaian Ikan dengan Menggunakan Model Pertumbuhan Logistik)

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ABSTRACT

This paper studies the harvesting strategies for tilapia fish farming. Two logistic growth models have been used namely constant harvesting and periodic harvesting. Even though tilapia fish farming has been commercialized, the use of mathematical models in determining harvesting strategies has not been widely applied in Malaysia. Logistic growth model is appropriate for population growth of animal when overcrowding and competition resources are taken into consideration. The objectives of this study were to estimate the highest continuing yield from fish harvesting strategies implemented. Secondly, the study predicted the optimum quantity for harvesting that can ensure the tilapia fish supply is continuous. Finally, to compare the results obtained between the two strategies. The best harvesting strategy for the selected fish farm is periodic harvesting. These findings can assist fish farmers to increase the supply to meet the demand for tilapia fish.

Keywords: Biomathematics; fisheries; harvesting; logistic growth model; periodic

ABSTRAK

Makalah ini membincangkan tentang strategi penuaian untuk penternakan ikan tilapia. Dua strategi menggunakan model logistik iaitu strategi penuaian tetap dan strategi penuaian berkala diketengahkan. Walaupun aktiviti penternakan ikan tilapia telah dikomersilkan, namun penggunaan model matematik dalam menentukan strategi penuaian berkesan tidak diaplikasikan di Malaysia. Strategi penuaian adalah sangat penting dalam membantu penternak ikan untuk membekalkan keperluan ikan tilapia kepada pengguna secara berterusan dan memastikan juga populasi ikan tersebut berada dalam keadaan yang stabil. Objektif utama kajian ini adalah untuk menentukan hasil penuaian yang berterusan daripada strategi penuaian yang digunakan. Seterusnya, kajian juga ingin menganggarkan jumlah optimum untuk penuaian ikan bagi memastikan sumber ikan dapat ditampung secara berterusan. Akhir sekali, keputusan daripada dua strategi penuaian ini akan dibandingkan. Strategi penuaian yang terbaik untuk penternakan ikan tilapia adalah penuaian berkala. Penemuan kajian ini adalah diharap akan dapat membantu para penternak ikan untuk meningkatkan bekalan ikan tilapia bagi memenuhi kehendak pengguna dan pasaran.

Kata kunci: Berkala; biomatematik; model pertumbuhan logistik; penuaian; perikanan

INTRODUCTION

Fish is one of the major sources of human diet and the main source of protein and fat. Recently, consumers have become more conscious of fish as a healthier alternative meat. This is particularly due to the problems with overweight and cardiovascular diseases that have turn into one of the major problems in human health. Awareness of fish as nutritious diet has caused the demand of fish for food consumption to increase. Supply of fish cannot rely only on the ocean fishing activities, thus, alternatives can be found by commercializing the agriculture.

Tilapia fish farming has been an important source in some areas of the world and it is well suited for farming since they are fast growing and hardy. Tilapia fish also can establish strong population in very short time duration if the environment is right (Gertjan et al. 2005). This has made tilapia fish a very important protein source.

Mathematical model have been used widely to estimate the population dynamics of animals for so many years as well as the human population dynamics. In recent years, the use of mathematical models has been extended to agriculture sector especially in cattle farming to ensure continuous and optimum supply. The logistic growth model in term of harvesting has been used to study the tilapia fish farming. According to Aanes et al. (2002), the most important for successful management of harvested populations is that harvesting strategies are sustainable, not leading to instabilities or extinctions and produces great results for the year with little variation between the years. Therefore, it can supply the market demand throughout the year.

Harvesting has been an area under discussion in population as well as in community dynamics (Murray 1993). Malthus was the first to formulate theoretical treatment of population dynamics in 1798 and Verhulst formed the Malthus theory into a mathematical model called the logistic equation that led to nonlinear differential equation (Alan 1992). Dubey et al. (2003), John et al. (2005) and Biswajit et al. (2007) agreed that it is a need to develop an ecologically suitable strategy for harvesting any renewable resource. Jing and Ke (2004a, 2004b), and Li and Wang (2010) studied the optimal harvesting policies as their management intention over a random harvesting time perspective.

According to Idels and Wang (2008), constant harvesting is where a fixed numbers of fish were removed each year, while periodic harvesting is usually thought of a sequence of periodic closure and openings of different fishing grounds. Harvesting has been considered a factor of stabilization, destabilization, improvement of mean population levels, induced fluctuations, and control of non-native predators (Michel 2007). Further reference on harvesting strategies can be found in Cooke and Nusse (1987) and Ludwig (2006).

In particular, fish farming in Malaysia has a great potential in economics contribution and supplying fish for food consumption. The use of mathematical model in harvesting fishes is conducted to help the fish farming sector and the Fisheries Department to estimate the population of tilapia fish for a given period. This will enable them to be prepared with effective solutions to ensure that the tilapia fish's supplies can fulfill the consumer demand.

The objective of this study was to estimate the highest continuing yield from fish harvesting strategies implemented. Secondly, the study recommended the optimum quantity of harvesting that can ensure that the tilapia fish supply is continuous. The final objective was to compare the results obtained between the two strategies. The strategies will ensure the supplies are continuous and the tilapia fish population stays is stable.

Methods

The data for this project has been obtained from the Department of Fisheries of Malaysia and from the fish owner of selected ponds suggested by the Department of Fisheries of Malaysia situated at Gombak, Selangor, Malaysia. The Department of Fisheries Malaysia (2008) claimed that a fish pond can sustain 5 tilapia fish for every 1 m^2 surface area. The selected pond has an area of 15.61 Ha, which is equivalent to 156100 m^2, the sustainable or carrying capacity, K of the pond is 780500 fish. The period of maturity for the tilapia fish is 6 months and estimates that 80% will survive to maturity (Thomas & Michael 1999). The Logistic Growth model can be written as:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right).$$

Here the variable P can be interpreted as the size of the population. Its development over time, $P(t)$ depends on its initial value $P(0)$ and on the two parameters r and K, where r is called the rate of fish survive at maturity stage and K is referred to as the carrying capacity of the population. Parameter H was introduced as harvesting function. Two types of harvesting strategy were developed as follows;

(i) The logistic growth model with constant harvesting;

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) - H(t).$$

(ii) The logistic growth model with periodic harvesting:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) - H(t),$$

where,

$$H(t) = \begin{cases} 156100 & , \ t \le 6 \\ 0 & , \ t > 6 \end{cases}$$

$$H(t + 12) = H(t)$$

Note that $H(t)$ is a periodic function of time with the period of one year. The fish population will not be able to extinct in fishing time since $H(t)$ is a periodic function and varies from season to season. The amount of fish might be able to increase again if in some season the fishing activity is stopped. The value of harvesting is approximated to be 156100 tilapia fishes by taking the summation value in the first six months.

Results and Discussion

The values of the parameters are $r = 0.8$, the estimation of fish that will survive at maturity stage and the value of carrying capacity, $K = 780500$. The equilibrium point is also called critical point or stationary point. At this critical point the fish population remains unchanged. The equilibrium points of the logistic growth model without fishing were obtained as shown:

$$\frac{dP}{dT} = 0,$$

$$rP\left(1 - \frac{P}{K}\right) = 0,$$

$$0.8P\left(1 - \frac{P}{780500}\right) = 0,$$

$$0.8P = 0,$$

$$P = 0,$$

$$1 - \frac{P}{780500} = 0,$$

$$\frac{P}{780500} = 1,$$

$$P = 780500,$$

This means that if the initial population started with $P = 0$, the population remains at $P = 0$. Similarly, if the initial population is started with $P = 5780500$, the population remains at the same level. The stability of this equilibrium points can be seen from Figure 1 where two values of an equilibrium points have been obtained.

Table 1 shows the intervals of equilibrium points that are showing whether the equilibrium point is stable or unstable. $P = 0$ is an unstable equilibrium point because the solutions near this point are repelled. This means given an initial population P_0 just above $P = 0$ and the P_0 less than 0, the population grows away from $P = 0$. The equilibrium point at $P_0 = 780500$ is a stable equilibrium point because solutions near this point are attracted to it. This means given an initial population in the interval (0, 780500), the population increases. If P_0 is greater than 780500, the population declines and approach a limiting values 780500.

LOGISTIC GROWTH MODEL WITH CONSTANT HARVESTING

The Logistic Growth Model with constant harvesting as follows:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) - H(t),$$

where the value of H is constant.

To determine the equilibrium points for H is constant:

$$0.8P\left(1 - \frac{P}{780500}\right) - H = 0,$$

$$0.8P - \frac{0.8P}{780500} - H = 0,$$

$$0.8P - 0.00000102498P^2 - H = 0.$$

By using square quadratic formula:

$a = 0.00000102498$
$b = -0.8$
$c = H$

$$P = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-0.8) \pm \sqrt{(-0.8)^2 - 4(0.00000102498)H}}{2(0.00000102498)}.$$

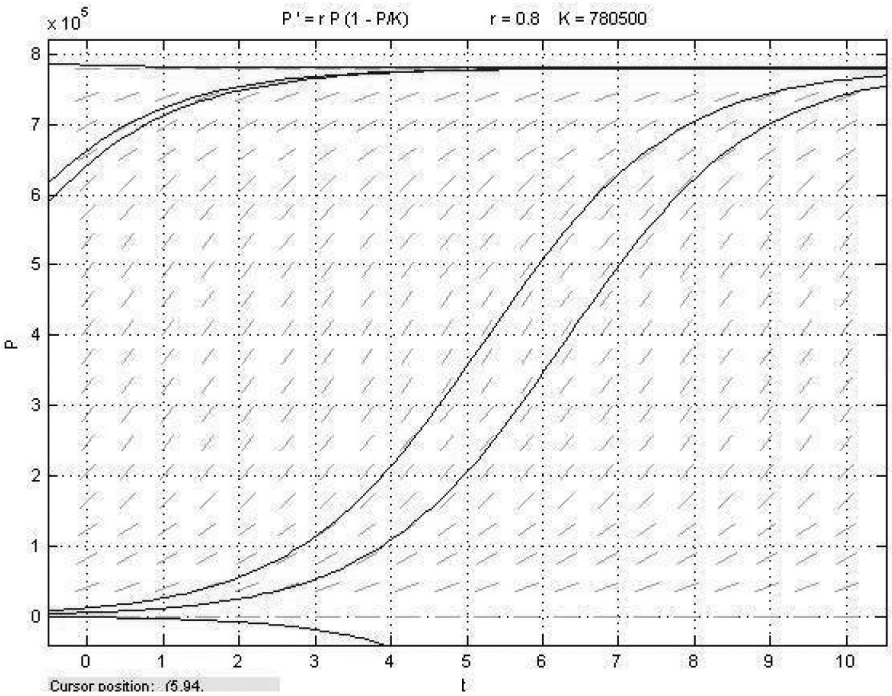

FIGURE 1. Constant Harvesting

TABLE 1. Interval of equilibrium points for logistic growth model

Interval	Sign of $f(P)$	$P(t)$	Arrow
$(-\infty, 0)$	Minus	Decreasing	Points Down
(0, 780500)	Plus	Increasing	Points Up
$(780600, \infty)$	Minus	Decreasing	Points Down

Consider the expressions under the square root sign. Letting this expression equals to 0, we have:

$(0.8)2 - 4(0.00000102498)H = 0$
$0.64 - 0.00000409992H = 0$
$H = 156100.6068 \approx 156100$

When the value of $H = 156100$ (known as a bifurcation point) then we consider 3 values of harvesting:

1. $H = 156100$
2. $H > 156100$
3. $H < 156100$

For $H = 156100$

From Figure 2, the value of harvesting, we have 1 equilibrium point. For P_0 larger than 389482; the population will decrease and approach to 389482. For P_0 less than 389482; the population will lead to extinction.

Table 2 shows the interval of equilibrium point that shows whether the equilibrium is stable or unstable point.

For $H > 156100$

From Figure 3, the value of harvesting is $H = 160000$ and this figure shows the decreasing trends of tilapia fish population. This implies the fish population will go to extinction regardless of the initial population size.

For $H < 15000$

From Figure 4, the value of harvesting, $H = 140000$. There are two equilibrium points exist when the value of harvesting is less than 156100. The upper equilibrium point is stable because the arrow in interval (515584, ∞) is grows down and it show that the population of fish is decreased. However, the arrow in interval (264919, 515584) grows up and show that the population of fish is increased. The lower equilibrium point is unstable because the solution near the point is repelled.

Table 3 shows that the interval of equilibrium points whether the equilibrium is stable or unstable points.

LOGISTIC GROWTH MODEL WITH PERIODIC FISHING

The ponds have full carrying capacity of 780500 tilapia fish in the ponds as an initial population (Figure 5). For the first six months 156100 tilapia fish is assumed for harvesting until the population of tilapia remains 515584 and followed by no harvesting for the next 6 months and this pattern repeats for several years. In order to ensure the population of tilapia fish is increasing, there are no harvesting in the next six months and the population of tilapia fish will increase until it approaches the carrying capacity, $K = 780500$.

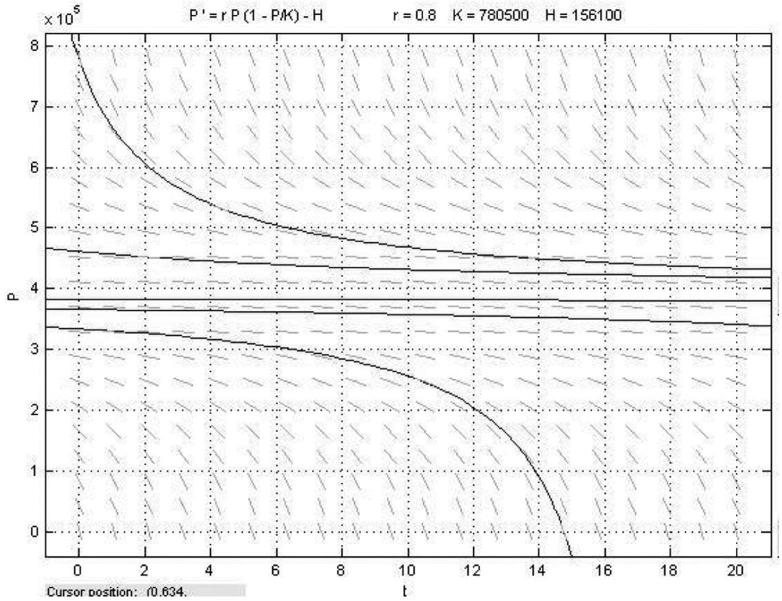

FIGURE 2. Harvesting = 156 100

TABLE 2. Interval of equilibrium points for harvesting = 156100

Interval	Sign of $f(P)$	$P(t)$	Arrow
(–∞, 389482)	Minus	Decreasing	Points Down
(389482, ∞)	Minus	Decreasing	Points Down

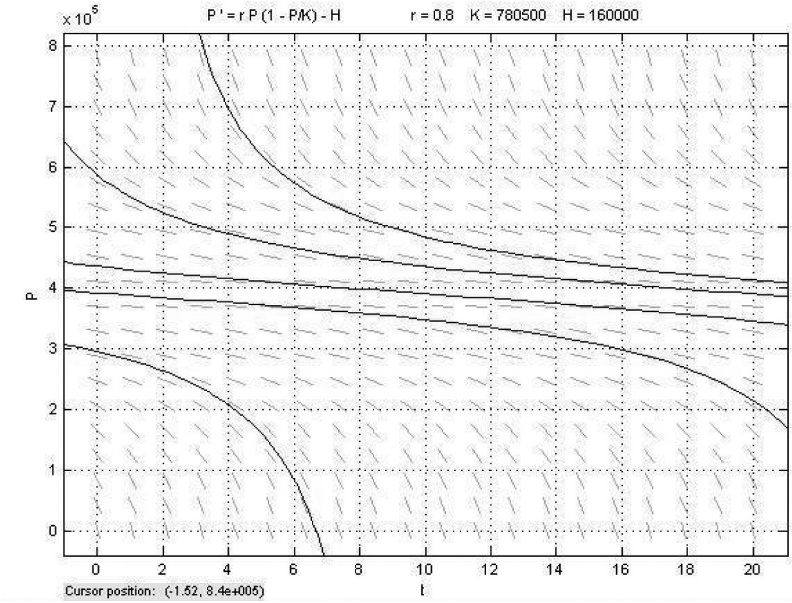

FIGURE 3. Harvesting = 160000

TABLE 3. Interval of equilibrium points for harvesting = 1 000 000

Interval	Sign of $f(P)$	$P(t)$	Arrow
$(-\infty, 264919)$	Minus	Decreasing	Points Down
(264919, 515584)	Plus	Increasing	Points Up
$(515584, \infty$	Minus	Decreasing	Points Down

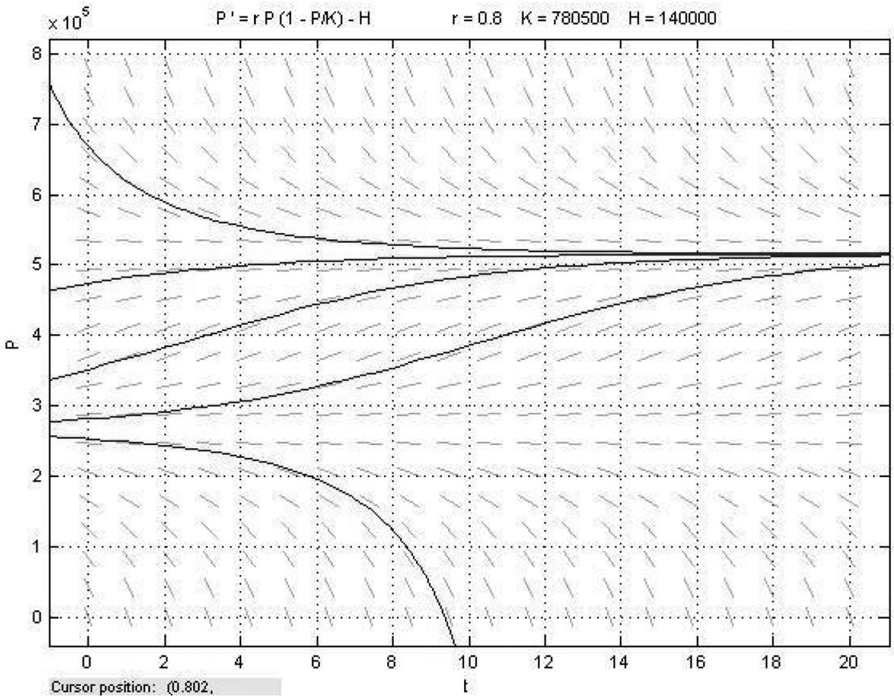

FIGURE 4. Harvesting = 140000

TABLE 4. Results from constant harvesting strategies

Constant Harvesting Strategies		
H = 156100	H < 156100	H > 156100
One equilibrium point P_0 = 389482	Two equilibrium points P_0 = 515584 P_0 = 264919	No points of equilibrium exist
Equilibrium point give the initial population.	Upper equilibrium point that gives the initial population is stale. Otherwise the, the lower equilibrium point gives the unstable initial population.	All the considered initial population values will lead to extinction.

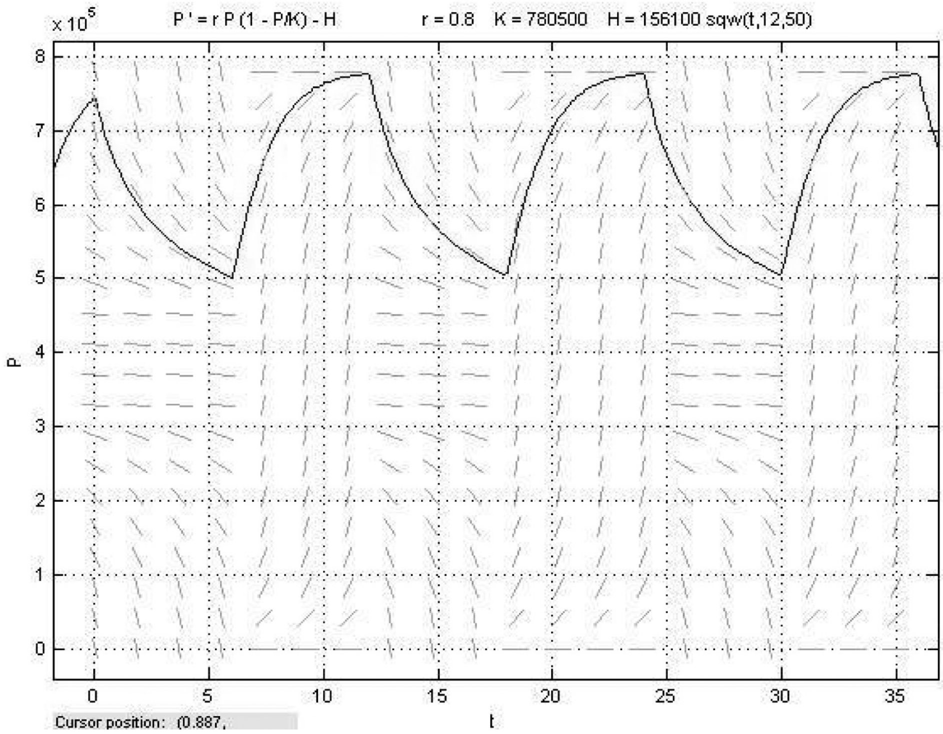

FIGURE 5. Logistic growth model with periodic fishing

CONCLUSION

The solution obtained optimizes the harvest while maintaining the stable population of fish is logistic periodic seasonal harvesting strategy. A harvesting strategy using logistic periodic seasonal harvesting strategy can be used to imporve productivity, shorten investment return time and reduce risk from changes in sale price and costs of productions, particularly when comparatively short return periods are used. However by using the constant harvesting, the fish farming does not have enough time to recover the fish population. The development of fish harvesting strategy probably can supply the market demand. It also can improve the commercial return to farmers before harvesting. This study can help in raising the fish such as tilapia fish in freshwater ponds for the farmer just like any other agricultural activity.

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